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CLUSTERING OF THE COSMIC RAY AGES OF STONE METEORITES

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A statistical analysis of the distribution of cosmic ray ages of stone meteorites indicates that significant discrete breakups probably need not be invoked to explain the apparent clustering of the hypersthene chondrites' ages. The clustering of the bronzite chondrites around 4 m.y. is not statistical.

1. INTRODUCTION

The place and manner in which the meteorites originated is not fully understood at present. Since the meteorites' cosmic ray ages are thought to be related to the elapsed time between their origin as distinct objects, and their fall to earth, the cosmic ray ages may be able to provide some information about the meteorites' origin. If an asteroidal origin of the meteorites is assumed, as it will be here, Kuiper's [1] result that catastrophic collisions between asteroids occur nearly continuously on a cosmic time scale suggests certain models for meteorite production and collection. Among these is one that hypothesizes that meteorites are created essentially continuously from asteroidal collisions, and are collected randomly by the earth.

This model has been criticized on the basis of visual inspection of the cosmic ray age data, particularly the cosmic ray ages of the hypersthene and bronzite chondrites. The objection is that a casual study of the cosmic ray ages reveals an apparent tendency for the ages to cluster, indicating that many meteorites were formed simultaneously as a result of a few significant breakups of parent objects. This interpretation of the cosmic ray age data tends to refute the continuous production, random collection model discussed above. To determine whether or not the apparent clustering can be used as an argument against the continuous production, random collection model, one must first find a quantitative way to measure the amount of clustering present in the data. Then one must determine how much clustering is to be expected from chance alone. Only if the observed clustering is greater than the amount reasonably attributable to normal statistical fluctuations can one infer discrete breakups in the meteorites' history on the basis of the cosmic ray ages. The question of whether the observed clustering is genuine or merely statistical is the subject of this paper.

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2. THE INTERVAL LAW

When periods of 10^6 or 10^7 years are considered, Poisson statistics can be used to describe the distribution of cosmic ray ages in time. From the Poisson distribution it follows that the probability of the earth capturing exactly x meteorites in a time t is:

$$P_t(x) = (at)^x \exp(-at)/x! \qquad (1)$$

where the mean capture rate is a meteorites per unit time. Hence the probability of having exactly x cosmic ray ages occur in a time t is given by equation (1).

If all the cosmic ray ages for a given class of meteorite were listed in ascending order, i.e. from most recent to oldest, the intervals between consecutive ages could be found by subtracting each age from the one following it. If data were available for K meteorites, $(K-1)$ such intervals would be found. These intervals measure the amount of clustering in the ages. If there were a lot of clustering, there would be many short intervals; conversely few short intervals would mean little clustering.

To determine whether or not the observed age clustering can be regarded as merely statistical, it is necessary to know the probability density

Table 1
Probability of the clustering being merely a statistical phenomenon

	Assumed uncertainty in the ages						
	0	±1%	±5%	±10%	±15%	±20%	±25%
Bronzite chondrites	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hypersthene chondrites	0.00	0.17	0.55	0.68	0.68	0.59	0.62

function for these intervals, that is, how should they be distributed, theoretically. If the earth's capturing a meteorite is regarded as an event, the probability of zero events during a time t follows from eq. (1):

$$P_t(0) = \exp(-at) \; . \tag{2}$$

Similarly, the probability of exactly one event between t and $t + \mathrm{d}t$ is:

$$P_{\mathrm{d}t}(1) = a\,\mathrm{d}t \tag{3}$$

to first order in infinitesimal quantities. Since the combination of no events for a time t and one event between t and $t+\mathrm{d}t$ is just an interval of length t, the function, $f(t)$, giving the fraction of intervals in the range t to $t+\mathrm{d}t$ is:

$$f(t)\,\mathrm{d}t = a\,\exp(-at)\,\mathrm{d}t \; . \tag{4}$$

It should be noted that more than 63% of the intervals are shorter than the mean interval, a; it is a general property of the Poisson distribution that rare events tend to cluster. Thus even if all meteorites were generated independently and collected randomly, there would still be a tendency for them to cluster, due to statistical rather than physical considerations. If K cosmic ray ages are measured, the expected number of intervals lying between t_1 and t_2 is:

$$N(t_1, t_2) = (K-1)(\exp[-at_1] - \exp[-at_2]) \, . \tag{5}$$

This result is commonly called the interval law.

3. THE CALCULATIONS

Since the expected distribution of intervals was known, from eq. (5), the chi-square test was used on the cosmic ray age data to see how well theory and observation agreed. Anders' [2] compilation of age data was used; most of the ages came from Kirsten et al. [3], and Hintenberger et al. [4]. The conventional 5% level of significance was used; if the data deviated from the expected values so much that only 5% of the time chance alone could account for the size of the fluctuation, the clustering was considered to be genuine, not statistical.

Because only a few of the ages were determined by measuring a radioactive nuclide and a stable nuclide in the same sample, there is reason to suspect that many of the ages may be slightly in error. Peaks in the distribution may have been smoothed out somewhat due to gas losses and various other uncertainties involved in the measuring process. To compensate for this, each published age was interpreted as meaning that the true age was a random variable distributed from a Gaussian population whose mean is equal to the published age. The standard deviation of this hypothetical population corresponds to the size of the assumed uncertainty in the data.

Thus each age was replaced by a random number drawn from the appropriate population. The chi-square test was applied to 300 sets of randomized age data (for each class of meteorite, separately) using as a standard deviation for each population, 1% of the population's mean. This represents a ±1% uncertainty in each age. The fraction of randomized data sets whose clustering could reasonably well be explained as due to chance alone was taken as the probability that the apparent clustering of the published ages is due solely to chance. The entire process was then repeated assuming uncertainties in the data of ±5%, ±10%, ±15%, ±20%, and ±25%.

From the results of these calculations, shown in table 1, three conclusions can be inferred. First, the clustering of the hypersthene chondrites' ages may well be statistical, and not reflect any significant breakups of a few parent objects. Second, the clustering of the bronzites around 4 million years is definitely not statistical. Third, the effects of error in the measured ages are very important.

4. SPACE EROSION

In 1959 Whipple and Fireman [5] proposed, and in 1966 Fisher [6] revived the idea that erosion in space may be the mechanism that makes the cosmic ray ages of the stone meteorites much shorter than those of the iron meteorites. The former's model predicts that a meteorite of true age t will have an apparent or measured age A related by:

$$A = T[1 - \exp(-t/T)] \; , \tag{6}$$

where T, the cutoff age, is a constant depending on the size of the meteorite and the erosion rate in space. If this were true, one would expect to find many meteorites of cosmic ray age close to but not exceeding some value, T. The only instance where the data resembles this sort of distribution is the case of the bronzites. Using this model, one can invert eq. (6) to find the true age, given the measured age and the cutoff:

$$t = -T \ln (1 - A/T) \ . \qquad (7)$$

These true ages should obey the interval law. To see if they do, the same Monte Carlo technique described in section 3 was used. Fifteen values of the cutoff age, evenly spaced from 3.0 to 6.0 m.y. were tried. Apparent ages greater than the cutoff were interpreted as true ages. These calculations indicate that the probability that an erosion dominated model would produce the observed data is considerably less than 0.10, even if the uncertainties in the ages were 25%.

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